# VAPOUR-LIQUID EQUILIBRIUM IN THE TERNARY SYSTEM CYCLOHEXANE-ACETIC ACID-PROPIONIC ACID 

Ivona Malijevská, Alena Maštalková and Marie Sýsová<br>Department of Physical Chemistry<br>Prague Institute of Chemical Technology, 16628 Prague 6

Received November 28, 1988
Accepted February 15, 1989

Dedicated to late Academician Eduard Hála.


#### Abstract

Isobaric equilibrium data ( $P=101.3 \mathrm{kPa}$ ) for the system cyclohexane-acetic acid-propionic acid have been measured by two different analytical techniques. Activity coefficients calculated by simultaneous solving of equations for the chemical and phase equilibria were subjected to a consistency test based on inaccuracies determined from the error propagation law, and were correlated by Wilson's equation. The activity coefficients measured were compared with those calculated from binary vapour-liquid equilibrium data and with values predicted by the UNIFAC method.


A large number of experimental binary vapour-liquid equilibrium data have been used in proposing methods for estimating activity coefficients and for determining the parameters for the various groups necessary for predicting the activity coefficients by the group contribution method. These methods have been developed primarily for estimating the activity coefficients of multicomponent mixtures ( $n>2$ ). Their verification is, however, frequently limited by the scarcity of experimental data for multicomponent systems.

The aim of the present work was to contribute to the data base and to study strongly associating systems.

## EXPERIMENTAL

Preparation of Pure Substances
All the chemicals used were supplied by Lachema Brno, and were purified before use as described below. Cyclohexane was freed from thiophene by shaking with concentrated sulphuric acid, pre--dried with $\mathrm{P}_{2} \mathrm{O}_{5}$, and distilled twice on a packed column. Acetic acid was purified by double distillation on a 40 -plate bubble-cap column. Propionic acid with added $\mathrm{KMnO}_{4}$ was distilled on a 20 -plate bubble-cap column furnished with a heated jacket.

The physicochemical properties of the compounds used in the measurements of the first nine
experimental points listed in Table II are given in Table I. In these measurements densities and refractive indices were used to determine the compositions of the liquid and vapour phases.

For the compounds used in the subsequent measurements, the purities as determined by gas chromatografic analysis using a flame ionization detector, which does not respond to water, were as follows: cyclohexane and propionic acid, $99.90 \%$; acetic acid, $99.92 \%$.

## Procedure

The circulation apparatus used to measure the vapour-liquid equilibrium is described elsewhere ${ }^{2}$.
The majority of experimental points for the compositions of the liquid and equilibrium vapour phases were obtained using a CHROM 61 gas chromatograph equipped with a flame ionization detector and a CI-100 integrator (Laboratorní přístroje, Prague). A glass column 990 mm in length and 2 mm in diameter was packed with SE-10 (Laboratorní přístroje, Prague) and operated isothermally at $150^{\circ} \mathrm{C}$. Both the injector and detector temperatures were $160^{\circ} \mathrm{C}$. Very good separation was achieved with nitrogen as the gas carrier. Calibration analyses were carried out to convert the peak area ratio to composition of the sample.

## RESULTS AND DISCUSSION

In order to obtain thermodynamically consistent activity coefficients, it is necessary to describe correctly the non-ideal behaviour of the components in the vapour phase. A model of a chemically reacting mixture was used, and the presence of the following six components in the vapour phase was assumed: cyclohexane monomer, monomers of both acids. dimers of both acids, and a mixed dimer (further referred to as true components).

In this case, the activity coefficient may be expressed as

$$
\begin{equation*}
\gamma_{i}(T, P)=\frac{f_{i 1}(T, P)}{x_{i} f_{i 1}^{*}\left(T, P_{(T)}^{0}\right)} \exp \left[1 / R T \int_{P}^{P_{i}^{\circ}} \mathscr{V}_{i}^{\circ 1} \mathrm{~d} P\right] \tag{1}
\end{equation*}
$$

where $T$ is the temperature, $P$ is the pressure, $x_{i}$ is the nominal mole fraction of $i$-th component in the liquid phase, $R$ is the gas constant, $f$ is the fugacity, and $\mathscr{V}_{i}$ is

Table I
Physicochemical properties of the compounds studied

| Compound | $n_{\text {D }}{ }^{0}$ |  | $d^{20}$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | measured | ref. ${ }^{1}$ | measured | ref. ${ }^{1}$ |
| Cyclohexane | 1.4262 | $1 \cdot 42623$ | 0.7784 | 0.77855 |
| Acetic acid | $1 \cdot 3717$ | $1 \cdot 3719$ | 1.0492 | 1.04926 |
| Propionic acid | 1.3864 | $1 \cdot 3865$ | 0.9933 | 0.9934 |

Table II
Vapour-liquid equilibrium data for the system cyclohexane(1)-acetic acid(2)-propionic acid(3)

| No. | $T, \mathbf{K}$ | $x_{1}$ | $x_{2}$ | $y_{1}$ | $y_{2}$ | $\gamma_{1} \pm s\left(\gamma_{1}\right)$ |  | $\gamma_{2} \pm s\left(\gamma_{2}\right)$ |  | $\gamma_{3} \pm s\left(\gamma_{3}\right)$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 360.52 | 0.327 | $0 \cdot 159$ | 0.789 | 0.085 | $2 \cdot 144$ | 0.03 | 1.427 | $0 \cdot 19$ | 1.215 | $0 \cdot 11$ |
| 2 | 361.36 | 0.255 | 0.247 | 0.738 | 0.123 | $2 \cdot 569$ | 0.05 | 1.211 | $0 \cdot 11$ | 1.255 | $0 \cdot 10$ |
| 3 | 364•78 | 0.183 | 0.424 | 0.672 | 0.226 | $3 \cdot 043$ | 0.09 | $1 \cdot 122$ | 0.06 | 0.994 | 0.12 |
| 4 | $358 \cdot 19$ | 0.320 | 0.351 | 0.741 | 0.168 | $2 \cdot 256$ | 0.04 | 1.245 | 0.08 | 1.350 | 0.17 |
| 5 | 354-81 | 0.609 | $0 \cdot 204$ | 0.811 | $0 \cdot 134$ | $1 \cdot 391$ | 0.01 | 2.052 | $0 \cdot 19$ | 1.754 | 0.37 |
| 6 | 353.79 | 0.772 | $0 \cdot 113$ | 0.863 | 0.106 | $1 \cdot 177$ | 0.01 | 3.378 | $0 \cdot 44$ | 1.862 | 0.69 |
| 7 | $361 \cdot 28$ | 0.289 | 0.245 | 0.754 | $0 \cdot 130$ | $2 \cdot 304$ | 0.04 | $1 \cdot 321$ | $0 \cdot 12$ | $1 \cdot 146$ | $0 \cdot 11$ |
| 8 | $355 \cdot 32$ | 0.656 | 0.133 | 0.857 | $0 \cdot 100$ | 1.317 | 0.01 | 2.579 | 0.33 | $1 \cdot 330$ | 0.35 |
| 9 | $363 \cdot 57$ | 0.344 | 0.062 | 0.828 | 0.046 | 1.923 | 0.03 | 2.000 | 0.55 | 1.046 | 0.09 |
| 10 | $384 \cdot 27$ | 0.051 | 0.398 | 0.378 | 0.348 | $4 \cdot 143$ | 0.41 | 1.060 | 0.04 | 1.017 | 0.05 |
| 11 | $386 \cdot 24$ | 0.035 | 0.503 | 0.293 | 0.455 | 4.606 | 0.66 | 1.032 | 0.03 | 1.042 | 0.06 |
| 12 | 387.55 | 0.023 | 0.681 | 0.205 | 0.616 | 5.014 | $1 \cdot 10$ | 0.987 | 0.02 | $1 \cdot 103$ | 0.09 |
| 13 | 387.21 | 0.018 | 0.762 | 0.180 | 0.685 | 5.752 | $1 \cdot 62$ | 0.985 | 0.02 | $1 \cdot 119$ | 0.12 |
| 14 | 383.60 | 0.021 | 0.851 | 0.216 | 0.715 | 6.504 | 1.60 | 0.988 | 0.02 | 1.087 | 0.21 |
| 15 | 371.97 | 0.058 | 0.822 | 0.417 | 0.539 | $5 \cdot 564$ | 0.49 | 1.016 | 0.02 | 1.000 | 0.28 |
| 16 | 379.67 | 0.047 | 0.640 | 0.365 | 0.485 | 4.981 | 0.54 | 0.996 | 0.03 | 1.084 | 0.10 |
| 17 | $370 \cdot 68$ | 0.088 | $0 \cdot 610$ | 0.503 | 0.389 | $4 \cdot 336$ | 0.25 | 1.054 | 0.03 | 1.057 | 0.13 |
| 18 | 362.17 | 0.173 | 0.544 | 0.627 | 0.295 | 3.311 | $0 \cdot 10$ | 1.150 | 0.05 | 1.083 | 0.17 |
| 19 | 357.83 | 0.284 | 0.476 | 0.684 | 0.250 | 2.441 | 0.05 | 1.283 | 0.06 | 1.260 | 0.23 |
| 20 | 367.45 | 0.164 | 0.339 | 0.648 | 0.201 | 3.057 | $0 \cdot 10$ | $1 \cdot 156$ | 0.07 | 1.071 | 0.09 |
| 21 | $375 \cdot 10$ | $0 \cdot 112$ | 0.259 | 0.586 | 0.184 | 3.359 | $0 \cdot 15$ | $1 \cdot 141$ | 0.08 | $1 \cdot 021$ | 0.05 |
| 22 | 377.49 | 0.046 | 0.754 | 0.380 | 0.538 | $5 \cdot 610$ | 0.62 | 0.982 | 0.02 | 0.972 | $0 \cdot 15$ |
| 23 | $366 \cdot 17$ | 0.104 | 0.711 | 0.533 | 0.411 | 4.965 | 0.24 | 1.061 | 0.03 | 0.997 | 0.22 |
| 24 | 374.93 | 0.080 | $0 \cdot 533$ | 0.497 | 0.359 | 4.204 | 0.27 | 1.024 | 0.04 | 0.989 | 0.09 |
| 25 | $355 \cdot 42$ | $0 \cdot 480$ | 0.227 | 0.791 | $0 \cdot 134$ | 1.707 | 0.02 | 1.748 | 0.15 | 1.460 | 0.22 |


the molar volume of $i$-th component. The subscript il denotes the monomer of $i$-th component, the subscripts 1 and o denote the liquid phase and pure component, respectively, and * denotes the value for the monomer of $i$-th component (for a non--associating component, $f_{i 1}^{*}=f_{i}^{\circ}$ ).

Since even for binary systems the corrections to the real behaviour of the true components are swamped by inaccuracies in the measured quantities, activity coefficients for this ternary system were calculated using a model of an ideally associating system, namely

$$
\begin{equation*}
\gamma_{i}=\frac{y_{i 1} P}{x_{i} y_{i 1}^{*} P_{i}^{o}} \exp \left[1 / R T \int_{P}^{P_{i}^{o}} \mathscr{V}_{i}^{o l} \mathrm{~d} P\right], \tag{2}
\end{equation*}
$$

where $y$ is the mole fraction of a component in the vapour phase.
In order to calculate the mole fractions of the monomers of the individual components in the vapour phase, mass balance equations (3), (4), and (5) must be solved simultaneously with chemical equilibrium equations (6), (7), and (8).

$$
\begin{gather*}
1=y_{\mathrm{C}}+y_{\mathrm{A} 1}+y_{\mathrm{A} 2}+y_{\mathrm{P} 1}+y_{\mathrm{P} 2}+y_{\mathrm{AP}}  \tag{3}\\
y_{\mathrm{A}}=\left(y_{\mathrm{A} 1}+2 y_{\mathrm{A} 2}+y_{\mathrm{AP}}\right) /\left(1+y_{\mathrm{A} 2}+y_{\mathrm{P} 2}+y_{\mathrm{AP}}\right)  \tag{4}\\
y_{\mathrm{P}}=\left(y_{\mathrm{P} 1}+2 y_{\mathrm{P} 2}+y_{\mathrm{AP}}\right) /\left(1+y_{\mathrm{A} 2}+y_{\mathrm{P} 2}+y_{\mathrm{AP}}\right)  \tag{5}\\
K_{\mathrm{A}}=y_{\mathrm{A} 2} P_{\mathrm{st}} / y_{\mathrm{A} 1}^{2} P  \tag{6}\\
K_{\mathrm{P}}=y_{\mathrm{P} 2} P_{\mathrm{s} 1} / y_{\mathrm{P} 1}^{2} P  \tag{7}\\
K_{\mathrm{AP}}=y_{\mathrm{AP}} P_{\mathrm{st}} / y_{\mathrm{A} 1} y_{\mathrm{P} 1} P \tag{8}
\end{gather*}
$$

The values for pure acids were obtained from Eqs (9) and (10).

$$
\begin{gather*}
1=y_{j 1}^{*}+y_{j 2}^{*}  \tag{9}\\
K_{j}=y_{j 2}^{*} P_{\mathrm{st}} \mid y_{j 1}^{* 2} P_{j}^{0}, \quad j=\mathrm{A}, \mathrm{P} \tag{10}
\end{gather*}
$$

where $K$ denotes the equilibrium constant of dimerization, the subscripts $\mathrm{A}, \mathrm{P}$, $A P$, and $C$ relate to acetic acid, propionic acid, mixed dimer, and cyclohexane, respectively, and the subscript st denotes the quantity in the standard state.

In the calculations we used the constants of Antoine's equation listed in Table III and the following temperature dependences of the equilibrium constants of dimerization (standard state: pure substance in the ideal gas state at a pressure of 1 Pa$)^{6.7}$

$$
\ln K_{\mathrm{A}}=7425.837 / T-29.2449
$$

and

$$
\ln K_{\mathrm{P}}=7635 \cdot 370 / T-29 \cdot 8390
$$

For the mixed dimer, we used the approximation

$$
K_{\mathrm{AP}}=2\left(K_{\mathrm{A}} K_{\mathrm{P}}\right)^{1 / 2}
$$

The measured vapour-liquid equilibrium data are listed in Table II, along with the calculated activity coefficients and their standard deviations.

In Fig. 1, the coexisting vapour and liquid compositions are connected by arrows pointing towards the vapour phase composition.

## I able III

Constants of Antoine's equation ${ }^{a}$

| Compound | $A$ | $B$ | $C$ |
| :---: | :---: | :---: | :---: |
| Cyclohexane $^{3}$ | 20.667105 | 2778.6003 | -49.917 |
| Acetic acid $^{4}$ | 23.939051 | 5199.7434 | 27.820 |
| Propionic acid $^{5}$ | 22.204615 | 3670.9490 | -70.545 |

${ }^{a} \ln P=A-B /(T+C)$, where $T$ is the thermodynamic temperature, and $P$ is in units of Pa .

FI. 1
Vapour-liquid equilibrium in the system cyclohexane(1)-acetic acid(2)-propionic acid(3); circles denote the liquid phase compositions, arrowheads the vapour phase compositions


The calculated activity coefficients were subjected to a consistency test in the form ${ }^{8}$

$$
\begin{equation*}
\sum_{i=1}^{3}\left(x_{i \mathrm{c}}+x_{i \mathrm{~d}}\right)\left(\ln \gamma_{i \mathrm{~d}}-\ln \gamma_{i \mathrm{c}}\right)=D \tag{II}
\end{equation*}
$$

where c and d denote a chosen pair of experimental points for which $\Delta x_{1}$ and $\Delta x_{2} \leqq$ $\leqq 0.1$ and $\Delta T \leqq 3 \mathrm{~K}$, and $D$ is the mean quadratic deviation obtained by the method of error propagation.

The calculation of the standard deviation in the activity coefficient was based on the following errors in the input quantities: $s\left(x_{1}\right)=s\left(y_{1}\right)=0.005, s\left(x_{2}\right)=s\left(y_{2}\right)=$ $=0.01, s(T)=0.05 \mathrm{~K}, s(P)=50 \mathrm{~Pa}, s\left(P_{i}^{\mathrm{o}}\right) / P_{i}^{\mathrm{o}}=0.003, s\left(K_{\mathrm{A}}\right) / K_{\mathrm{A}}=0.05, s\left(K_{\mathrm{P}}\right) /$ $/ K_{\mathrm{P}}=0 \cdot 1$, and $s\left(K_{\mathrm{AP}}\right) / K_{\mathrm{AP}}=0 \cdot 1$.

No suitable partners for the consistency test were found within the above defined ranges of $\Delta x_{1}, \Delta x_{2}$, and $\Delta T$ for points No. $9,11,14,15,20,21$, and 22 ; the other experimental points passed the test.

The activity coefficients were correlated with composition by the Wilson equation

$$
\begin{equation*}
\ln \gamma_{i}=-\ln \sum_{s=1}^{3} x_{s} A_{i s}+1-\sum_{r=1}^{3}\left(x_{r} \Lambda_{r i} / \sum_{s=1}^{3} x_{s} A_{r s}\right) \tag{12}
\end{equation*}
$$

where

$$
\begin{equation*}
\Lambda_{i j}=\mathscr{V}_{j}^{0} / \mathscr{V}_{i}^{0} \exp \left(-k_{i j} / T\right) \tag{13}
\end{equation*}
$$

The objective function was the sum of weighted squares of the deviations in the logarithms of activity coefficients. In this way, we obtained the following set of constants:

$$
\begin{array}{lll}
k_{12}=318.833 & k_{13}=44.137 & k_{23}=546.617 \\
k_{21}=822.554 & k_{31}=811.009 & k_{32}=-326.378
\end{array}
$$

The standard deviation of the correlation

$$
\begin{equation*}
s=\left(\sum_{i=1}^{N} \sum_{j=1}^{3} w_{i j}\left(\ln \gamma_{i j}^{\exp }-\ln \gamma_{i j}^{\mathrm{cal}}\right)^{2} /(3 N-6)\right)^{1 / 2} \tag{14}
\end{equation*}
$$

was $s=1 \cdot 2$, where $N$ is the number of experimental points, $w$ is the weight, and the superscripts cal and exp denote the calculated and experimental values, respectively.

By calculating the vapour composition and temperature with the use of the constants of the Wilson equation as obtained by the ternary data correlation, we found the average errors $\Delta \bar{y}=0.007$ and $\Delta \bar{T}=0.6 \mathrm{~K}$. The same back calculation using constants of the Wilson equation obtained from binary data ${ }^{9}$ yielded the average errors $\Delta \bar{y}=0.009$ and $\Delta \bar{T}=1.9 \mathrm{~K}$.

A better agreement with experimental data than one might have expected was obtained by the use of the UNIFAC prediction method, which yielded the average errors $\Delta \bar{y}=0.01$ and $\Delta \bar{T}=0.6 \mathrm{~K}$.

## REFERENCES

1. Riddick J. A., Bunger W. B.: Organic Solvents. Wiley, New York 1970;
2. Malijevská I., Sýsová M., Vľ̌ková D.: Collect. Czech. Chem. Commun. 51, 194 (1986).
3. Willingham G. J., Taylor W. J., Pignocco J. M., Rossini F. D.: J. Res. Natl. Bur. Stand. (U.S.) 35, 219 (1945).
4. Wichterle I., Linek J.: Antoine Vapor Pressure Constants of Pure Compounds. Academia, Prague 1971.
5. Ambrose D., Ghiassee N. B.: J. Chem. Thermodyn. 19, 505 (1987).
6. Bartoñ J. R., Hsu C. C.: J. Chem. Eng. Data 14, (2), 184 (1969).
7. Taylor M. D., Bruton J.: J. Am. Chem. Soc. 74, 4151 (1952).
8. McDermott C., Ellis S. R. M.: Chem. Eng. Sci. 20, 293 (1965).
9. Gmehling J., Onken U., Arlt W.: Vapor-Liquid Equilibrium Data Collection, Dechema Chemistry Data Series, Vol. I, Part 5. Dechema, Dortmundt 1980.

Translated by M. Škubalovà.

